ARTICLE VII.

Investigations which led to the detection of the coincidence between the computed place of the Planet Leverrier, and the observed place of a Star recorded by Lalande, in May, 1795. *By Sears C. Walker. Read February* 19*th*, 1847.

Washington, D. C., February 13*th*, 1847.

To Dr. Robert M. Patterson:

My Dear Sir,—I hasten to comply with the invitation in your letter of yesterday to lay before the American Philosophical Society the steps that led to the detection of the very remarkable coincidence between the computed place of the planet Leverrier and the observed place of a star of the 7.8 magnitude, which passed the meridian of Paris at 14*h*. 11*m*. 23*s*.5 of Lalande's clock time, May 10th, 1795.

Soon after the arrival of the news of the *physical* discovery of Leverrier, a suggestion, by Mr. E. C. Herrick, of its possible identity with the Wartman planet of 1831, induced me to commence the search for the approximate elements of the two. I soon came to the conclusion that Leverrier could not have been in Wartman's region in 1831, and that no satisfactory orbit could be found for Wartman's planet, from the imperfect tracing of its path in the Contes Rendus for 1836.

In this first inquiry concerning the motions of Leverrier, I learned the near approach of its orbit to the circular form. The same analogies of the solar system that furnished Leverrier and Adams the clue to their *analytical* discovery of the planet, were to be the guides in the first attempt to sketch its orbit.

It was naturally to be presumed that the inclination and eccentricity of this primary planet were small, and that with a radius vector nearly twice that of Herschel, the sun's power to impress daily variations of the radius vector, and orbital motion must be comparatively small, and that in a first approximation these daily variations, as well as their first and second differences, might be neglected.

Accordingly, I commenced with the simple hypothesis of *constancy* of the *radius vector*, from September 26th, to November 21st, a period of fifty-eight days, leaving the character of the orbit, whether nearly circular or much flattened, to be the result of the investigation.

Nine European observations combined, furnished me one place of Leverrier, September 26, 1846. Three Washington observations, October 24th, and three more, November 21st, completed the three observed places. I commenced with the trial of radii vectores 33 and 34, which include Leverrier's and Adams's hypothetical values. I found that 33 was too great, and extended the scale downwards to 32, 31, 30, and 29.

I subjoin the table of computed daily sidereal orbital motions for the three intervals, of thirty days from September 26; fifty-eight days from September 26; and twenty-eight days from October 24, for this scale of assumed constant radii vectores. They are the results of an *approximate* computation only. r is the radius vector, n' is the daily sidereal orbital motion for the first thirty days, n for the whole term of fifty-eight days, and $n_{,}$ for the last twenty-eight days. μ is the mean daily sidereal motion for $r = a =$ the semi-axis major.

r	n'	n	$n_{,}$	μ
34	12″.8	16″.7	19″.7	17″.90
33	14 .6	17 .7	20 .3	18. 71
32	16 .6	18 .8	20 .8	19. 60
31	19 .4	20 .1	21 .2	20. 56
30	21 .7	21 .6	21 .6	21. 58
29	24 .1	23 .4	22 .0	22. 67

The same analogies that led to the assumption of the *constancy* of the radius vector for fifty-eight days, also lead to the conclusion that n must be nearly constant. Accordingly, the true radius vector, to be interpolated from this table, was that in which $[(n - n')^2 + (n - n_{,})^2]$ should be a minimum.

A slight inspection of the table shows that this value of r is very nearly 30.0, and that since for this value $n' = \mu$, $n = \mu$, and $n_{,} = \mu$, very nearly, therefore $a = r$ very nearly. In other words, the orbit approaches very nearly to the circular form, and that which was at first inferred from the analogies of the solar system (viz., the smallness of the eccentricity,) is now established as a deduction from actual observation.*

I was now prepared to commence a rigorous computation of the *circular* elements, on the hypothesis of a uniform radius vector, $r = a$. For this computation two observed places are sufficient. I chose the above place of the 26th of September, and a place deduced from my own observations with the Washington Equatorial, December 26th. On this night I compared Leverrier in right ascension by transits, thirty-three times, with each of the two Enckian stars which have been used for comparison with Leverrier from its discovery by Galle, to the middle of January last. I also compared it eleven times in declination with the same stars, with the filar micrometer.

A test of the precision of a night's work with the equatorial is furnished by the fact that the observed relative position of the two fixed stars should be constant on all the nights. In this way I found the probable error of one night's work to be about 0″.6 of space in Diff. R. A., and 0″.5 in Diff. Dec. I mention this to show the precision of the measured path when many nights' works are combined, and the comparisons are throughout made with the same stars. After correcting the observed places for parallax, and

* The possible accidental case of $r = a$ in a very eccentric ellipse, was rejected from its improbability.

the dates for aberration time, (which amounts to about four hours,) I computed the (I.) elements in the table below. The data for the computation were obtained as follows. The planet's mean place as a fixed star for January 1st, 1847, was derived from the observations. The correction for planetary parallax was applied. The R. A. and Dec. were then reduced to their equivalent geocentric latitude and longitude (α and δ,) referred to the mean equinox of January 1st, 1847, with the mean obliquity. The places are,

*Greenwich mean time,	θ, θ'	=	$1846^y\ 268^d.33333$	;	$1846^y\ 359^d.5$
Aberration time,	$\triangle\theta, \triangle\theta'$	=	$-0^d.16755$	;	$-0\ .17563$
Reduced time,	t, t'	=	" $268^d.165783$	;	" $359^d.32437$
Planet's Geo. Lon.,	α, α'	=	325°49′ 1″.48	;	326° 4′ 2″.64
Planet's Geo. Lat.,	δ, δ'	=	—0°31′57″.81	;	—0°31′26″.37
Concluded hel. Lon.,	λ, λ'	=	326°59′ 5″.40	;	327°31′59″.00
Concluded hel. Lat.,	β, β'	=	—0°31′ 7″.089	;	—0°32′ 4″.791
	$\lambda'-\lambda$, $\beta'-\beta$	=	+0°32′53″.60	;	+0° 0′57″.702
Orbital Longitudes,	v, v'	=	326°59′32″.73	;	327°32′27″.09
True motion, in whole term, $v-v'$		=	0°32′54″.36	;	
$\frac{v'-v}{t'-t}$ =	n,	=	21″.658575	;	
For $r = a = 29.93995$,	μ	=	21″.658575	;	

The smallness of the values of $\lambda'-\lambda$, and $\beta'-\beta$, on which the concluded position of the plane of the whole orbit depends, would have deprived this result of all its value, if the errors of observation had not also been extremely small, so as to bear a corresponding proportion to the measured path. There is, however, one advantage accompanying this smallness of $v'-v$, viz., that the errors arising from the neglected terms, (the daily variations of r and n,) are more nearly insensible.

With the elements (I.) I computed with every possible precision an ephemeris of Leverrier from August 1st, 1846, to February 1st, 1847, and then compared with it all the standard observations yet received, after applying all the small corrections, and treating them in the same manner as the places of September 26th and December 26th, above.

The available observations comprehend one hundred and sixteen nights' works in all. They may be thus classified.

MERIDIAN OBSERVATIONS.

No. of nights observed.	Observatory.	Instrument.	Observer.
5	Göttingen,	Meridian Circle,	Gauss.
13	Altona,	"	Petersen.
17	Hamburg,	"	Rumker.
4	Dorpat,	"	Madler.
4	Konigsberg,	"	Wichtman.
3	Geneva,	"	Plantamour.
2	Turin,	"	Plana.
8	Cambridge, E.,	"	Challis.
5	Washington,	Transit Inst.,	Almy.
4	"	"	Keith.
6	"	Mural Circle,	Coffin.
1	"	"	Page.
3	"	Ertel. Merid. Circle,	Maynard.
5	"	"	Hubbard.

* The earth's *true* place is taken out from the time θ, and θ'. The earth's latitude was taken into account.

DIFFERENTIAL MEASURES.

No of nights observed.	Observatory.	Instrument.	Observer.	Star of Comparison.
10	Berlin,	Great Equatorial,	Encke and Galle,	α of Encke.
6	Cambridge, E.,	" "	Challis,	7648 B. A. C.
7	Washington,	" "	Maury,	7648 " "
11	"	" "	Walker,	7648 " "
4	"	" "	Hubbard,	7648 " "

The place of the star seventh magnitude 7648 B. A. C. rests upon the following authorities. Piazzi, Mayer, Taylor, Challis, six observations; Plantamour, four observations; Plana, one observation; Washington observatory, twenty-five observations. The above are direct observations of 7648 B. A. C. There are also ten Berlin, and three Washington equatorial comparisons of 7648 B. A. C. with Leverrier directly or through ✱ *a*, on nights when Leverrier was observed on the meridian, and its place was reduced to a common date.

The other star, Encke's ✱ *a*, ninth magnitude, has for its place the following authorities, Bessel's Zones, Encke and Galle, two nights' comparisons with 7648 B. A. C., Encke, five nights' comparisons with Leverrier referred on these nights to fifteen meridian observations; Maury, six; Walker, ten nights' comparisons with 7648 B. A. C.

I have adopted the mean place of the two stars from all these authorities for January 1st, 1847, as follows:

7648 B. A. C.	7th mag.,	R. A. 1847,	327°32′16″.79	December, 1847,	—13°23′39″.17
✱ *a* of Encke,	9th mag.,	"	327°57′42″.81	"	—13°25′57″.22

I do not think that the error of either of these star's places much exceeds one second of space.

By means of my ephemeris I was able to compute the value of *c*, or the mean of the second differences of the planet's daily places in R. A. and Dec. with a certainty of an error not exceeding 0″.02. The group of observations of any seven consecutive nights were reduced to the corresponding value for the fourth night, by the following formula, in which the differences *c* of the daily motions, and not of the motions themselves, are employed.

$$\text{I. Normal place (4th night,) } \alpha = \frac{1}{\Sigma\, h\, h}\Big[\begin{array}{ll} h\ h & \left(\alpha_0\right) \\ h'\ h' & \left(\tfrac{1}{2}\alpha_{-1} + \tfrac{1}{2}\alpha_{+1} - \tfrac{1}{2}c\right) \\ h''\ h'' & \left(\tfrac{1}{2}\alpha_{-2} + \tfrac{1}{2}\alpha_{+2} - \frac{2\times 2}{2}c\right) \\ h'''\ h''' & \left(\tfrac{1}{2}\alpha_{-3} + \tfrac{1}{2}\alpha_{+3} - \frac{3\times 3}{2}c\right)\Big] \end{array}$$

Where

$$\begin{aligned} n &= \text{the number of nights' works combined for single night.} \\ h &= \surd n \\ h' &= \surd\left(4, \frac{n_{-1} \times n_{+1}}{n_{-1} + n_{+1}}\right) \\ h'' &= \surd\left(4, \frac{n_{-2} \times n_{+2}}{n_{-2} + n_{+2}}\right) \\ h''' &= \surd\left(4, \frac{n_{-3} \times n_{+3}}{n_{-2} + n_{+3}}\right) \end{aligned}$$

In this manner I obtained from the above list of observations of Leverrier sixteen normal places, which I subjoin, together with the corrections of the ephemeris. In this list α and δ are the mean places of Leverrier as a fixed star in latitudes and longitudes referred to the mean equinox and mean obliquity of January 1st, 1847, corrected for planetary parallax, but not corrected for planetary aberration.

NORMAL PLACES OF LEVERRIER.

No. of Place.	Mean Time, Greenwich.	Obs. Geo. Lon. α	No. of Obs.	Obs. Geo. Lat. δ		Obs. Eph. $\Delta\alpha$	Obs. Eph. $\Delta\delta$
1	1846 215d.56696	328° 9′49″.34	(1)	— 0°31′36″.24	(1)	— 16″.75	— 0″.63
2	223 54405	327 57 9 .04	(1)	44 .09	(1)	— 7 .27	— 1 .03
3	270 5	325 46 25 .82	(16)	57 .99	(16)	— 1 .02	+ 0 .84
4	276 5	39 54 .23	(13)	56 .14	(13)	+ 0 .27	+ 1 .51
5	282 5	34 16 .11	(13)	56 .09	(13)	+ 1 .12	+ 0 .03
6	290 5	28 21 .99	(12)	53 .16	(12)	+ 3 .13	+ 0 .80
7	298 5	24 25 .25	(18)	51 .13	(19)	+ 4 .19	+ 0 .56
8	306 5	22 32 .46	(6)	47 .61	(6)	+ 3 .02	+ 0 .23
9	313 5	22 40 .00	(4)	45 .15	(3)	+ 2 .40	— 0 .68
10	319 5	24 6 .41	(4)	41 .51	(6)	+ 1 .95	+ 0 .51
11	325 5	26 50 .59?	(4)	37 .30?	(4)	+ 3 .77?	+ 2 .21? *rej.*
12	334 5	23 9 .44	(7)	33 .92	(6)	+ 2 .46	— 1 .13
13	345 5	44 26 .93	(4)	30 .79	(4)	+ 0 .96	— 0 .03
14	353 5	54 58 .01	(2)	27 .10	(2)	— 0 .72	+ 1 .51
15	359 5	326 4 2 .54	(3)	26 .04	(3)	— 0 .23	+ 0 .77
16	372 5	326 26 39 .11	(3)	23 .60	(3)	— 4 .40	+ 1 .28

A slight examination of the corrections of the ephemeris from Elements (I.) deviates slightly though sensibly from the circular form. Accordingly, the next step in the investigation was to remove the restriction $r = a$, $n = \mu$, and merely suppose the radius vector constant during the observed interval, and leaving n to take such a value as the observations should require.

For this purpose let $x = 50 \times \Delta r$
$y = 10 \times \Delta v$
$z = \Delta \lambda_{300}$ = correction of hel. lon. by Eph., October 29th, 1846.
$v =$ daily motion in hel. lon.

From the sixteen normal places nine equations of condition were formed, with equal weights. No. 11 was rejected. Equation 1 is the third of the mean of Nos. 1 and 2. Equations 2, 3, 4, 5 and 6, are Nos. 3, 4, 5, 6 and 7, respectively. Equation 7 is the mean of Nos. 8, 9 and 10; Equation 8 of Nos. 12 and 13; Equation 9 of Nos. 14, 15 and 16.

After reducing the correction of the geocentric to those of heliocentric longitudes and latitudes, and computing the coefficients of x and y (that of z is always 1,) the nine conditional equations from the latitudes were,

						Outstanding Error.
1 ;	$0 = -0.303 \times x$	$-$ 2.700 $\times y$	$+$ 0.333 $\times z$	$+ 3''.88$	;	$- 0''.08$
2 ;	$= + 3.016$	$-$ 3.000	$+$ 1.	$+ 1 .00$	;	$+ 0 .49$
3 ;	$= + 3.363$	$-$ 2.400	$+$ 1.	$- 0 .27$	;	$+ 0 .19$
4 ;	$= + 3.685$	$-$ 1.800	$+$ 1.	$- 1 .10$	;	$+ 0 .22$
5 ;	$= + 4.038$	$-$ 1.000	$+$ 1.	$- 3 .07$	;	$- 1 .03$
6 ;	$= + 4.268$	$-$ 0.200	$+$ 1.	$- 4 .12$	;	$- 1 .31$
7 ;	$= + 4.594$	$+$ 1.267	$+$ 1.	$- 2 .44$	;	$+ 1 .03$
8 ;	$= + 4.248$	$+$ 3.950	$+$ 1.	$- 1 .73$	;	$- 0 .13$
9 ;	$= + 3.332$	$+$ 6.133	$+$ 1.	$+ 1 .81$	;	$- 0 .16$

Whence the three normal equations,

$$\begin{array}{llllll} 0 = & 118.879 \times x & + \ 7.477 \times y & + \ 30.443 \times z & - \ 45''.629 \\ 0 = & 7.477 & + \ 85.149 & + \ 0.250 & + \ 1 .687 \\ 0 = & 30.443 & + \ 0.250 & + \ 8.111 & - \ 8 .627 \end{array}$$

And

$$\text{II.} \left\{ \begin{array}{llll} x = + 3 .255712 \ , & r = & 29.939950 + \frac{x}{50} & = 30 .005064 \\ y = - 0''.272963 \ , & n = & \frac{v'-v}{t'-t} & = 21''.65789 \\ z = - 11 .1475 \ , & v \text{ Sept. 28th, 1846} & & = 326°59'34''.74 \end{array} \right.$$

The values of n and v are the result of a new computation with the new radius vector 30.005064. The sum of the squares of the errors in heliocentric longitude of Elements (I.) for the nine equations is 55″.96. The sum of the similar quantities for Elements (II.) is 4″.21, which is the sum of squares of nine errors, each of which is composed of the united errors of theory and observation.

In my paper on meteors in the Memoirs of the American Philosophical Society, New Series, Vol. VIII., I have given the well-known equations,

$$\text{III.} \qquad \frac{1}{a} = \frac{2}{r} - g^2 = \frac{2}{r} - \left(\frac{rn}{\text{Gauss's } x}\right)^2$$

$$\text{IV.} \qquad er \ \cos v = a \cos^2 \phi - r = a\,(1-e^2) - r$$

In which g is the true orbital velocity in units of the earth's mean orbital velocity. Equation (III.) by means of r and n in Elements (II.) gives $a = 30.200585$. Equation IV. gives the value of v for any assumed value of the eccentricity. It is of the second degree and gives the value of either in the first or fourth quadrant. It is not possible from the process above pursued, to decide between the two quadrants of v. By hypothesis the daily variation of r was neglected. Hence it remains uncertain whether the r of Elements (II.) belongs to the first or fourth quadrant.

It is possible that the insertion in the conditional equations of two more terms for the daily variations of r and n might decide this point. Before attempting this inquiry I resolved to examine the ancient catalogues for the purpose of detecting Leverrier as a missing star.

Among the catalogues to be resorted to were Bradley's, Lacaille's, Mayer's, Lalande's, Piazzi's, Bessel's, Brisbane's and Taylor's. The first three of these catalogues do not usually include stars of the 7 . 8th magnitude. In the recent publication of Piazzi's original observations by the Vienna Observatory, extending from 1792 to 1798, I do not find among the stars observed by Piazzi and not afterwards identified, any which came

within reasonable limits of Leverrier's computed place on the nights of observation. From 1821 to 1832, the term of Bessel's zone observations, Leverrier was near the southern point of the ecliptic, and consequently below Bessel's limit. Brisbane's catalogue is not by me at present, and Taylor's observations at Madras are usually confined to the reviewing of stars in previous catalogues. There remained only Lalande's catalogue which offered hopes of success at present. A sketch of the Leverrier regions for several periods from 1790 to 1800, soon showed that there were but two nights in which I could expect to find observations of Leverrier in the Histoire Céleste, viz., those of the 8th and 10th of May, 1795. The corrections of the clock and quadrant for these two nights are nearly the same. Accordingly, I made an approximate computation of Leverrier's place on the latter night from my Elements (II.,) using the present radius vector and present orbital motion, viz., 21″.6 ± 0″.3. This limit appeared to me sufficiently extensive to include the probable place. After reducing the computed R. A. and Dec. from the mean equinox of January 1st, 1847, to the apparent place for May 10th, 1795, and then applying the reductions to Lalande's clock time and recorded zenith distance, I found by this approximate computation the *locus* of the Leverrier region, May 10th, 1795, thus,

	Clock time of transit.	Quadrant reading.	
Leverrier if in H. C. * 7.8 mag.;	13*h*. 59*m*. 2*s*.;	59° 6′ 7″;	supposed western limit.
* 7.8 ;	14 5 17 ;	59 37 21 ;	probable place.
* 7.8 ;	14 11 32 ;	60 8 35 ;	supposed eastern limit.

All the stars observed in this region on the 8th of May, 1795, were below the 7.8 magnitude, and were found in Bessel's Zones.

The only star in this region on the 10th reads thus:

	Clock time of transit.	Quadrant reading.
Histoire Celeste. * 7.8 mag.;	14*h*. 11*m*. 23*s*. 5 ;	60° 7′ 19″

I was at once struck with the coincidence in quadrant reading of this star with that part of the *locus* of Leverrier which has 14*h*. 11*m*. 23*s*.5 for its clock time, and which has for its quadrant reading 60° 7′ 50″. I examined Bessel's 242d Zone, comprising the same region. The Lalande star was not there.

This computation and comparison with the H. C. was made on the evening of the 2d of February, a cloudy night. I extended the limits to those which would result from ± 0″.6 of difference of average orbital motion since 1795. Still there was no other star in the H. C. which could have been supposed to be Leverrier. The only other star in this region of double extent, not found in Bessel's Zones was entered by Lalande as of the ninth or tenth magnitude.

I immediately drew up a statement of my conviction that the star 7.8 mag., 14*h*.11*m*.23*s*.5 of the H. C. of May 10th, 1795, was Leverrier, and that on search for it the next clear night it would therefore be missing. My confidence was such that I furnished this statement to Lieut. Maury, and submitted a copy made for my private use to my friends Prof. Hubbard, Prof. Coffin, Lieut. Gillis, Prof. Bache, and Prof. Henry. It is proper to add, that both Prof. Hubbard and Prof. Coffin, who were familiar with all the steps of the inquiry, expressed their strong belief that the star would be missing on an appeal to the

heavens. I furnished Prof. Hubbard with the list of guide-stars from Hussey's XIVth Hour of the Akademischen Sternkarten, as follows:

Mag. 9 ,	R. A. 1800, 14*h*. 8*m*. 1*s*.,	Dec. 1800, — 11°39′.2,	Authority B_1	
9 ,	8 3 ,	— 11 27.5	B_1	
8 ,	8 5 ,	— 11 8.0	B_2	
8 ,	10 0 ,	— 11 26.5	L_2 B_1	
9 ,	10 27 ,	— 10 55.1	B_1	
8 ,	10 33 ,	— 10 28.4	L_1 B_1	
9 ,	11 27 ,	— 10 53.3	L_1	
8 ,	12 0 ,	— 11 8.3	L_1 B_1	
7.8,	12 1 ,	— 11 21.0	L_1	{ Expected to be missing as being Leverrier.
6 ,	12 41 ,	— 10 47.6	B_2 P	
9 ,	13 20 ,	— 11 25.8	B_1	
7 ,	13 57 ,	— 10 45.2	B_2 P	
9 ,	15 53 ,	— 10 42.9	B_1	
8.9,	20 32 ,	— 10 58.4	L_1 B	

The first clear night after detecting this coincidence was the fourth of February. Prof. Hubbard examined the region with the Equatorial, and reported to Lieut. Maury the next morning, February 5th, that he had found all the guide-stars in this list; but that the star that was designated as *expected to be missing*, was indeed *missing*.

Prof. Hubbard reviewed the region several times. The star which should have preceded the missing star by 1° was in place. It was brought to the middle transit wire and to the bottom of the field. The Lalande star should have been in the upper portion of the field. It was not there. Nor was there in the vicinity any star that could be reasonably supposed to have been erroneously recorded as in the place of the missing star. I may add, that this region has since been examined by Lieut. Maury and Prof. Hubbard. The star is certainly *missing*.

On the hypothesis that this missing star was Leverrier, I have computed the (III.) Elements below. They show that on this supposition Leverrier is now approaching the perihelion, or, in other words, is in the fourth quadrant of true anomaly.

I submit the three sets of elements above referred to:

Elements of Leverrier, for mean time Greenwich referred to the mean Equinox of January 1st, 1847.		Circular Hypothesis. I.	Elliptic Hypothesis. II.	Elliptic Elements if identical with the missing star of Lalande's H. C. III.
Longitude of the perihelion	π	Unknown	Unknown	0° 12′ 25″.51
Longitude of the ascending node	☊	129° 48′ 23″.16	129° 48′ 23″.16	131° 17′ 35″.80
Epoch of mean longitude, Jan. 1st, 1847	M	Unknown	Unknown	328° 7′ 56″.64
True longitude on the orbit, Sep. 28th, 1846	ω	326° 59′ 41″.50	326° 59′ 34″.74	326° 59′ 34″.74
Radius vector, Sep. 28th, 1846	r	29.93995	30.005064	30.02596
Daily sidereal orbital motion, Sep. 28th, 1846	n	21″.658575	21″.65789	21″.64553
Inclination	i	1° 45′ 19″.88	1° 45′ 19″.88	1° 54′ 53″.83
Eccentricity	e	0	Unknown	0.0088407
Mean distance	a	29.9395	30.200585	30.25042
Mean daily sidereal motion	μ	21″.658575	21″.37881	21″.32600
Period in tropical years	T	163ʸ.8259	165ʸ.9703	166ʸ.38134

The question of their identity will be decided in due course of time. If confirmed, this observation of Lalande will be exceedingly precious to the astronomer in discussing the question of the existence of other planets superior to Leverrier.

I have remarked, that the above estimate of the limits of the Leverrier region for May 10th, 1795, was based on only approximate computations. I expected to examine all the stars in this region with the Equatorial, and if any were missing that were seen by Lalande, then to investigate the question of their identity with Leverrier. The coincidence in place with the Lalande star which induced me to believe beforehand that on examining the heavens it would be missing, was quite unexpected. I have stated the particulars that led to this singular coincidence. Now that the star is known to be missing, I will proceed to examine the plausibility of the hypothesis of their identity.

The first objection to this hypothesis is, the shortness of the period compared with the *hypothetical* periods obtained by Adams and Leverrier (first published by the latter,) from the equations of condition derived from the residuary perturbations of Herschel. To this objection it may be answered, that both of those analytical discoverers of Leverrier assumed as the basis of their research a mean distance double that of Uranus, and then only diminished its value as the distortion of the other resulting elements compelled them to do so. Hence the extraordinary eccentricity in their first results, $\frac{16}{100}$ by Adams, and $\frac{11}{100}$ by Leverrier. Had they allowed free scope to the variations of the mean distances instead of forcing the other elements to conform to the preconceived value, I doubt not that a shorter period and more nearly circular orbit would have resulted. In proof of this remark I will quote from Mr. Adams's letter of September 2d, 1846, to the Astronomer Royal, (see pages 529 and 530 of the London and Edinburgh and Dublin Philosophical Journal, No. 197, for December, 1846.)

"*St. John's College, Cambridge, September 2d,* 1846.

"In the investigation, the results of which I communicated to you last October, the mean distance of the supposed disturbing planet is assumed to be twice that of Uranus. Some assumption is necessary in the first instance, and Bode's law renders it probable that the above distance is not very remote from the truth: but the investigation could scarcely be considered satisfactory while based on any thing arbitrary; and I therefore determined to repeat the calculation, making a different hypothesis as to the mean distance. The eccentricity also resulting from my former calculations was far too large to be probable; and I found that although the agreement between theory and observation continued very satisfactory down to 1840, the difference in subsequent years was becoming very sensible, and I hoped that these errors, as well as the eccentricity might be diminished by taking a different mean distance. Not to make too violent a change, I assumed this distance to be less than the former value by about one-thirtieth part of the whole. The result is very satisfactory, and appears to show that *by still farther diminishing the distance, the agreement between theory and the later observations may be rendered complete, and the eccentricity reduced at the same time to a very small quantity.* The mass and the elements of the orbit of the supposed planet, which result from the two hypotheses, are as follows:

	Hypothesis I $\left(\frac{a}{a'} = 0.5\right)$	Hypothesis II. $\left(\frac{a}{a'} = 0.515\right)$
Mean longitude of the planet 1st October, 1846,	325°8′	323°2′
Longitude of the Perihelion	315°.57	292°.11
Eccentricity	0.16103	0.12062
Mass, (that of the sun being 1)	0.00016563	0.00015003

"The investigation has been conducted in the same manner in both cases, so that *the differences between the two sets of elements may be considered as wholly due to the variation of the fundamental hypothesis.*

"The errors given by the Greenwich observations of 1843, are very sensible, being, for the first hypothesis + 6″.48, and for the second, + 5″.50. By comparing these errors, it may be inferred that the agreement of theory and observation would be rendered very close by assuming $\frac{a}{a'} = 0.57$, and the corresponding mean longitude on the 1st of October would be about 315° 20′, which I am inclined to think is not far from the truth. *It is plain also that the eccentricity corresponding to this value of* $\frac{a}{a'}$ *would be very small.*"

This letter of Mr. Adams's is exceedingly valuable in the present inquiry. His most probable value of $\frac{a}{a'} = 0.57$ gives 33.6842 for the mean distance. The variation of the eccentricity, according to Adams's Elements I. and II. for a variation of about one-thirtieth of the primitive mean distance of 38.4, in $\frac{4141}{16103}$ths of the eccentricity when the primitive value is 0.16103. From this proportion between the variations of the mean distance and eccentricity, the value of $e_{,}$ of the latter may be derived from any assumed value $a_{,}$ of the former by the formula

$$\text{V.} \qquad e_{,} = 0.16103 \times \left[\frac{0.12062}{0.16103}\right]^{\left(\frac{\log \frac{a_{,}}{38.4}}{\log \frac{1}{1.03}}\right)}$$

With the mean distance $a_{,} = 30.200585$ of my Elements II., formula V. gives for the eccentricity $e_{,} = 0.01538825$. I am thus enabled to complete Elements II. without any assumption respecting the missing star of Lalande, with one deficiency only, and that is the want of knowledge of the fact whether the radius vector is now increasing or diminishing. The complete elements for the only two possible cases are,

Elements of Leverrier, in which the mean distance is derived from actual *observation*, and the *eccentricity* from the mean distance, by means of the ratio between them computed by Adams from the observed residual perturbation of Herschel, referred to the mean equinox of January 1st, 1847, and to mean noon, Greenwich.		CASE I. for radius vector now *increasing* ELEMENTS II.	CASE II. for do. *diminishing* ELEMENTS II.
Longitude of the perihelion,	π	261° 4′ 14″.20	32° 54′ 55″.28
" ascending node,	☊	129° 48′ 23″.16	129° 48′ 23″.16
Inclination,	i	1° 45′ 19″.88	1° 45′ 19″.88
Mean distance,	a	30.200585	30.200585
Epoch of mean longitude, January 1st, 1847,	M	326° 33′ 59″.93	328° 32′ 51″.53
Eccentricity,	e	0.0153883	0.0153883
Period in tropical years,	T	165ʸ.9703	165ʸ.9703
Mean daily sidereal motion,	μ	21″.37881	21″.37881

With these elements which are the result of observed motions of Leverrier, and of observed residual perturbations of Herschel, I have computed Leverrier's place for May 10th, 1795, and reduced this place to the apparent equinox of that date, and to the actual condition of Lalande's clock and quadrant. The results are,

VI.	May 10th, 1795, Leverrier as ✱ 7.8 mag.	14*h*. 3*m*. 14*s*.81 ;	59° 26′ 47″;	Case I.
		14 14 10 .48 ;	60 23 20 ;	Case II.

It appears at once, on comparison with these computed places, that there is no star in the H. C. near the first which is not also in Bessel's Zones. The second place, for Case II., points at once to the remarkable star 7.8 mag., 14*h*. 11*m*. 23*s*.5; 60° 7′ 19″.

In order to determine the quadrant reading which the computed place would have from Elements II., Case II., if we vary the eccentricity so as to make the ephemeris give, for Lalande's clock time, 14*h*. 11*m*. 23*s*.5, I subjoin the locus of Leverrier, May 10th, 1795, for Case I. and Case II., and for eccentricities varying within the probable limit of that element. These limits I have taken as follows,

Minimum limit for $v = 0$, $r = 30.005064$, $e = 0.006474$
Maximum limit greater than that of Jupiter, Saturn, or Uranus, $e = 0.06$

ELEMENTS II.
Locus of Perihelion of Leverrier, and R. A. and Dec., May 10th, 1795, referred to mean equinox of 1800, for comparison with catalogues.

	e	π	*R. A.*	*D.*
Case I.	0.06	239° 45′ 23″.3	13*h*. 45*m*. 50*s*.	— 9° 3′.1
"	0.05	241 34 53 .6	49 30	— 9 25 .7
"	0.04	244 2 17 .0	53 51	— 9 47 .8
"	0.03	247 46 18 .2	57 45	— 10 8 .1
"	0.02	254 47 30 .3	14 1 41	— 10 29 .1
"	*0.0153883	261 4 15 .2	3 52 .2	— 10 40 .40
"	0.01	276 54 7 .1	6 21 .9	— 11 9 .45
"	0.0064740	326 59 34 .7	9 18 .0	— 11 8 .75
Case II.	0.01	17 5 2 .4	12 9 .1	— 11 23 .46
"	*0.0153883	32 54 55 .3	14 47 .9	— 11 36 .97
"	0.02	39 11 39 .1	16 36	— 11 44 .5
"	0.03	46 12 51 .3	20 35	— 12 6 .3
"	0.04	49 56 52 .5	24 28	— 12 30 .8
"	0.05	52 24 15 .9	28 4	— 12 43 .2
"	0.06	54 13 46 .2	32 8	— 13 2 .6

The position of the missing star of Lalande for 1800 is,

	R. A. 1800.	Dec.
✱ 7.8 mag.	14*h*. 12*m*. 0*s*.9	— 11° 20′ 58″
Leverrier by interpolation,	14 12 0 .9	— 11 22 56
Eccentricity by interpolation,	0.009875	

The rigorous computation does not present so close a coincidence as my first approximation, February 2d. The place by interpolation is suited to the actual radius vector

* Value for Elements II. from formula V.

for the eccentricity 0.009875, whereas that of February 2d was made with the radius vector of September 28th, 1846, supposed unchanged.

The preceding table of the locus of π for the various values of e presents a striking confirmation of the other arguments in favour of the supposition of a small eccentricity. For the limits of e the table gives,

For,	$e = 0.06$,	$\pi = 239°.8$		Case I.
			$+ 37°.1$	
	$e = 0.01$,	$\pi = 276.9$		Case I.
			$+ 50.1$	
	$e = 0.006474$,	$\pi = 327.0$		Case I. or II.
			$+ 50.1$	
	$e = 0.01$,	$\pi = 17.1$		Case II.
			$+ 37.1$	
	$e = 0.06$,	$\pi = 54.2$		Case II.

If now, we suppose, in the original creation of the system, all points of the orbit as having equal probability of being perihelion points; and from the analogies of the system that 0.06, is the probable limit of the value of e, and that each degree of the orbit has a unit of prŏbability (a priori) of being the perihelion point, we have for the number of such degrees within the above limits,

$$P = 37°.1 + 50°.1 + 50°.1 + 37°.1 = 174°.4$$

Calling $p_{e'-e}$ = the probability that the eccentricity should fall within the limits of e' and e we have from loci derived from my Elements II.

$$p_{0.01-0.006474} = \frac{100.2}{174.4} = 0.5745$$

$$p_{0.06-0.01} = \frac{74.2}{174.4} = 0.4255$$

From which it appears that there are 575 chances in 1000 for the eccentricity to fall between 0.006471 and 0.01; and only 425 chances in 1000 for the eccentricity to be greater than 0.01. If Leverrier was the missing star, the eccentricity by Elements III. was 0.0088407, which is near the middle point of the most probable limit.

I beg leave, in conclusion, to remark, that after a careful examination of all the circumstances known to me at this time, I find none that militates against the hypothesis that Leverrier was the missing star of Lalande.

I subjoin a table of all the stars in the Histoire Céleste situated within 15′ of the locus of Leverrier, (in declination in the above table,) reduced to their mean places for 1800.

No.	Mag.	R. A. 1800	Dec. 1800	
1	9.10	13h.50m.36s.	— 9° 24′.0	L_1
2	7.8	13 52 48	— 9 58.8	L_1
3	7.8	13 52 53	— 9 45.7	L_1, B_1
4	8.9	13 57 13	— 10 11.7	L_1, B_1
5	9	13 59 54	— 10 26.4	L_1, B_1
6	8	14 10 0	— 11 26.5	L_1, B_1
7	8	14 12 0	— 11 8.3	L_1, B_1
8	7.8	14 12 09	— 11 20.96	L_1, missing ✳
9	8	14 29 37	— 13 10.7	L_1, B_1

With the exception of the missing star, Nos. 1 and 2 are the only ones not excluded from the hypothesis of identity with Leverrier, by having been since found in Bessel's Zones. No. 1 I reject as too small, and No. 2 I reject as too far south of the computed place. No. 8 was then, before examining the heavens, the only plausible candidate for being missing.

It is possible that the computation of the perturbations for May 10th, 1795, may set aside this hypothesis, though I think it improbable.

Yours, very respectfully,

SEARS C. WALKER.